

A FEW REMARKS ON MIXING PROPERTIES OF C^* -DYNAMICAL SYSTEMS

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ABSTRACT. We consider strictly ergodic and strictly weak mixing C^* -dynamical systems. We prove that the system is strictly weak mixing if and only if its tensor product is strictly ergodic, moreover strictly weak mixing too. We also investigate some other mixing properties of the system.

1. Introduction. It is known [13, 15] that a notion of mixing for dynamical systems plays an important role in quantum statistical mechanics. A lot of papers, see, [5, 6, 9, 10, 16], were devoted to the investigations of mixing properties of dynamical systems. Very recently in [11] certain relations between ergodicity, weak mixing and uniformly weak mixing conditions of C^* -dynamical systems have been investigated. It is known [8, 16] that strict ergodicity of a dynamical system is stronger than ergodicity. Therefore, it is natural to ask how this notion is related with mixing conditions. The object of this paper is to investigate this question. Namely, we are going to consider strictly ergodic and strictly weak mixing C^* -dynamical systems. The paper organized as follows. In Section 2 we recall some preliminaries on C^* -algebras and dynamical systems. Section 3 is devoted to the characterization of strictly ergodic C^* -dynamical systems. In Section 4 we prove that the system is strictly weak mixing if and only if its tensor product is so. We also introduce a notion of ϕ -ergodicity and compare it with known mixing conditions.

2. Preliminaries. In this section we recall some preliminaries concerning C^* -dynamical systems.

Let \mathfrak{A} be a C^* -algebra with unit $\mathbf{1}$. An element $x \in \mathfrak{A}$ is called *self-adjoint*, respectively *positive*, if $x = x^*$, respectively there is an element

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