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GLOBAL ATTRACTORS FOR CROSS DIFFUSION SYSTEMS ON DOMAINS OF ARBITRARY DIMENSION

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ABSTRACT. A general triangular cross diffusion system given on a domain of arbitrary dimension n is considered. It will be shown that (L^∞, L^p) boundedness implies uniformly boundedness. The general result is then applied to several systems to obtain global existence. In some cases, the existence of a global attractor is also proven.

1. Introduction. Ever since the fundamental work by Amann, see [**2**–**5**], there has been much interest in the study of strongly coupled parabolic systems. The question of local existence of solutions was settled by Amann's work but global existence results seem to be answered in only very few cases.

In this paper we will consider a class of triangular cross diffusion systems given on an open bounded domain Ω in $\mathbb{R}^n$ with $n \geq 1$. Let us consider quasilinear/linear differential operators

$$\mathcal{A}_u(u,v) = \nabla(P(x,u,v)\nabla u + R(x,u,v)\nabla v),$$
$$\mathcal{A}_v(v) = \nabla(Q(x,v)\nabla v) + c(x)v,$$

and the following parabolic system

$$\begin{cases} \partial u/\partial t = \mathcal{A}_u(u,v) + g(u,v) & x \in \Omega,\ t > 0, \\ \partial v/\partial t = \mathcal{A}_v(v) + f(u,v) & x \in \Omega,\ t > 0, \end{cases} \tag{1.1}$$

with mixed boundary conditions for $x \in \partial\Omega$ and $t > 0$

$$\begin{cases} \chi(x)[(\partial v/\partial n)(x,t) + \alpha(x)v(x,t)] + (1-\chi(x))v(x,t) = 0, \\ \chi(x)[(\partial u/\partial n)(x,t) + \beta(x)u(x,t)] + (1-\chi(x))u(x,t) = 0, \end{cases} \tag{1.2}$$

where χ is a given function on $\partial\Omega$ with values in $\{0,1\}$. The initial conditions are described by

$$v(x,0) = v^0(x), \quad u(x,0) = u^0(x), \quad x \in \Omega \tag{1.3}$$

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