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ON THE NUMBER OF SUBSEQUENCES WITH GIVEN SUM OF SEQUENCES OVER FINITE ABELIAN p-GROUPS

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ABSTRACT. Let G be an additive finite abelian p-group. For a given (long) sequence S over G and some element $g \in G$, we investigate the number of subsequences of S which have sum g. This refines some classical results of J.E. Olson and recent results of I. Koutis.

1. Introduction and main result. Let G be an additively written finite abelian group. The enumeration of subsequences of a given (long) sequence over G, which have some prescribed properties, is a classical topic in combinatorial number theory going back to P. Erdős, J.E. Olson, et al. In the meantime there is a huge variety of results achieved by many authors, see [**1–6, 8–11, 14–16**] and the literature cited therein, for an overview of the various types of results.

In this note we concentrate on finite abelian p-groups. In order to state our main result, we need some notations, for details see Section 2. Suppose that $G = C_{n_1} \oplus \cdots \oplus C_{n_r}$, where $1 < n_1 \mid \cdots \mid n_r$ and set $\mathrm{d}^*(G) = \sum_{i=1}^r (n_i - 1)$. For a sequence S over G, an element $g \in G$ and some $k \in \mathbf{N}_0$, let $\mathrm{N}_g(S)$ ($\mathrm{N}_g^+(S)$, $\mathrm{N}_g^-(S)$, respectively $\mathrm{N}_g^k(S)$) denote the number of subsequences T of S having sum g (and even length, odd length respectively, length k).

Theorem 1.1. *Let G be a finite abelian p-group, $g \in G$, $k \in \mathbf{N}_0$ and $S \in \mathcal{F}(G)$ a sequence of length $|S| > k \exp(G) + \mathrm{d}^*(G)$.*

1. $\mathrm{N}_g^+(S) \equiv \mathrm{N}_g^-(S) \mod p^{k+1}$.

2. *If $p = 2$, then* $\mathrm{N}_g(S) \equiv 0 \mod 2^{k+1}$.

3. *If $j \in [0, \exp(G) - 1]$ and $m^* = k - 1 + \lceil (1 + \mathrm{d}^*(G))/\exp(G) \rceil$, then the numbers $\mathrm{N}_g^{m \exp(G)+j}(S)$ for all $m > m^*$ are modulo* $\mod p^k$ *uniquely determined by $\mathrm{N}_g^j(S)$, $\mathrm{N}_g^{\exp(G)+j}(S), \ldots, \mathrm{N}_g^{m^* \exp(G)+j}(S)$.*

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