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H-CONTACT UNIT TANGENT SPHERE BUNDLES

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ABSTRACT. We study how the geometry of a Riemannian manifold (M, g) is influenced by the property that its unit tangent sphere bundle $(T_1M, \eta, \bar{g})$ is *H-contact*, that is, the characteristic vector field ξ of T_1M is harmonic.

1. Introduction. The study of the geometric properties of a Riemannian manifold (M, g) via the investigation of its unit tangent sphere bundle T_1M, is a well known and interesting research field in Riemannian geometry. T_1M can be equipped with its "natural" metric g_S (the one induced by the Sasaki metric of the tangent bundle), as well as with the contact metric $\bar{g}$ of its standard contact metric structure $(\eta, \bar{g})$. In both cases, geometrical properties of T_1M influence those of the base manifold M itself, and conversely.

For example, all the information about the geodesics of (M, g) is encoded in the geodesic flow on T_1M, which is precisely the characteristic vector field ξ of its standard contact metric structure $(\eta, \bar{g})$. Riemannian manifolds whose unit tangent sphere bundle is either K-contact or (strongly) φ-symmetric or a (k, μ)-space, were completely classified, see [**3, 8, 23**]. We can refer to [**12**] for a survey about the contact metric geometry of T_1M.

Recently, many authors have studied the harmonicity of unit vector fields in several geometric situations, see, for example, [**14**] for a survey. If (M, g) is a compact and orientable Riemannian manifold, a unit vector field V of M is called *harmonic* if it is a critical point for the energy functional restricted to the set of all unit vector fields of M, [**24, 25**].

An interesting geometrical situation, in which a distinguished vector field appears in a natural way, is given by a contact manifold (M, η) where we have the characteristic vector field ξ. On the other hand, ξ

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