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AN ELEMENTARY PROOF OF A THEOREM CONCERNING THE DIVISION OF A REGION INTO TWO

OLGA BODROŽA-PANTIĆ, RADE DOROSLOVAČKI
AND KSENIJA DOROSLOVAČKI

ABSTRACT. Intuitively obvious theorems which are hard to prove are nothing new in topology. The most celebrated case is certainly the Jordan curve theorem. For pedagogical reasons elementary proofs of such theorems never become obsolete. During their work with students of mathematics the following problem has forced itself on the authors of the paper: to prove in a reasonably elementary fashion that an open Jordan curve with its endpoints on a closed Jordan curve $\mathcal{K}$, but otherwise located in the bounded part, divides the closure of the bounded part into two parts. In this paper we take the Jordan curve theorem (JCT) for granted and then prove, in a careful, elementary way, the related fact. Unfortunately, it seems that, even given the JCT, there is still a whole lot of work to do. But there are shortcuts. For instance, we do not need to consider the problem of approximating a general curve by polygons, or the delicate limit questions arising when going back from the easy polygon case to the general case.

1. Introduction. The Jordan curve theorem (JCT) claims that a simple closed curve in a plane divides the plane (excluding the points of the curve $\mathcal{K}$ itself) into two regions in the sense that any broken line (curve consisting of connected line segments) connecting two points from different regions intersects the curve, and for any two points from the same region there exists a broken line connecting them which does not intersect the curve. Exactly one of these regions is bounded and called the *interior*; the other one is called the *exterior* of the curve. A (bounded) figure Φ determined by a simple closed curve $\mathcal{K}$ is usually defined as the union of the curve $\mathcal{K}$ and its interior. Using the fact that the curve $\mathcal{K}$ is the boundary of each of its regions, proved in [**1**], one readily obtains that the interior and the boundary of Φ, denoted by In(Φ) and Bd(Φ), coincide with the interior of $\mathcal{K}$ and the curve $\mathcal{K}$ itself, respectively.

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