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GENERALIZED PRESCRIBED SCALAR CURVATURE TYPE EQUATION ON A COMPACT MANIFOLD OF NEGATIVE SCALAR CURVATURE

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ABSTRACT. This paper deals with the problem of the so-called generalized prescribed scalar curvature type equation on a compact Riemannian manifold with negative scalar curvature. We give the existence of a positive solution which is the subject of the first theorem. In the second one, we prove the multiplicity of solutions of the subcritical quasilinear elliptic equation.

1. Introduction. Let (M,g) be a Riemannian n-manifold. For $n \geq 3$, if $g' = u^{4/(n-2)}g$, $u \in C^\infty(M)$, $u > 0$, on M, is a metric conformal to g, the scalar curvatures R and $\widetilde{R}$ of g and g' respectively satisfy the equation

$$\Delta_g u + \frac{n-2}{4(n-1)} R u = \frac{n-2}{4(n-1)} \widetilde{R} u^{2^*-1}$$

where $2^* = (2n/n-2)$ and $\Delta_g u = -\mathrm{div}_g(\nabla u)$ is the Laplacian of u.

A smooth function f on M will be the scalar curvature of a conformal metric g' if there exists a function $u \in C^\infty(M)$, $u > 0$, solution of the equation

$$(1) \qquad \Delta_g u + \frac{n-2}{4(n-1)} R u = f u^{2^*-1}.$$

Such equation has been intensively studied in the past two decades: as examples, we can refer to the works of Aubin [**1**], Bahri-Coron [**2**], Escobar-Schoen [**4**], Hebey [**6**], Kazdan-Warner [**7**], Schoen [**9**] and Druet [**3**].

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