ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 37, Number 4, 2007

THE REAL GENUS OF GROUPS OF ODD ORDER

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ABSTRACT. Let G be a finite group. The *real genus* $\rho(G)$ is the minimum algebraic genus of any compact bordered Klein surface on which G acts. Here we consider groups of odd order acting on bordered surfaces. First we show that if G is a group of odd order, then the real genus $\rho(G)$ is even. We also obtain a stronger result for p-groups. Let p be an odd prime, and let G be a p-group with $\rho(G) \geq 2$; then the real genus $\rho(G) \equiv p+1 \bmod 2p$. We also examine "large" automorphism groups of odd order. If the odd order group G acts on a bordered Klein surface of genus $g \geq 2$, then $|G| \leq 3(g-1)$. If G acts with the largest possible order $3(g-1)$, then we call G an *O^*-group*. In general, a quotient Q of an O^*-group G is again an O^*-group, and a surface X on which G acts is a full covering of a surface of lower genus on which Q acts. Thus, it is natural to consider the notion of an *O^*-simple* group, that is, an O^*-group with no O^*-quotient. We classify the O^*-simple groups.

1. Introduction. Let G be a finite group. The *real genus* $\rho(G)$ is the minimum algebraic genus of any compact bordered Klein surface on which G acts. A *real genus action* of G is an action of G on a bordered surface of (algebraic) genus $\rho(G)$. There are now several results about the real genus parameter. The groups with real genus $\rho \leq 8$ have been classified [**7, 12, 13, 16**], and genus formulas have been obtained for several families of groups [**13–16**]. In particular, McCullough determined the real genus of each finite abelian group [**21**].

Here we consider groups of odd order acting on bordered Klein surfaces. Our main result is the following.

Theorem 1. *If G is a group of odd order, then the real genus $\rho(G)$ is even.*

We also obtain a stronger result for p-groups.

2000 AMS *Mathematics Subject Classification.* Primary 30F50, Secondary 20H10, 57M60.

Received by the editors on June 18, 2004, and in revised form on March 22, 2005.