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COMMON INVARIANT SUBSPACES FOR FINITELY QUASINILPOTENT COLLECTIONS OF POSITIVE OPERATORS ON A BANACH SPACE WITH A SCHAUDER BASIS

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ABSTRACT. We prove that if $\mathcal{C} \neq \{0\}$ is a collection of continuous positive operators on a Banach space with a Schauder basis that is finitely quasinilpotent at a nonzero positive vector, then $\mathcal{C}$ and its positive commutant $\mathcal{C}'_+$ have a common nontrivial invariant closed subspace.

In 1995, Y.A. Abramovich, C.D. Aliprantis and O. Burkinshaw [**4**] showed that every continuous positive operator S on a Banach space X with a Schauder basis which commutes with a nonzero continuous positive operator T on X that is quasinilpotent at a nonzero positive vector has a nontrivial invariant closed subspace. In this paper, using the Abramovich-Aliprantis-Burkinshaw technique based on the idea from [**2, 4**], we extend the result to a collection $\mathcal{C}$ of operators on X and obtain the result that $\mathcal{C}$ and its positive commutant $\mathcal{C}'_+$ have a common nontrivial invariant closed subspace. In particular, all continuous positive operators on a Banach space X with a Schauder basis which commute with a nonzero continuous positive operator T on X that is quasinilpotent at a nonzero positive vector have a common nontrivial invariant closed subspace.

In order to do this, we first recall some of the basic terminologies and facts from [**4, 5**] and others. For the notions and facts not stated in the text we refer to [**1**–**15**] and so on.

In this note, the word *operator* will be synonymous with *linear transformation*. Let X be a Banach space and $B(X)$ the Banach

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