ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 37, Number 4, 2007

PHILOS-TYPE OSCILLATION CRITERIA FOR SECOND ORDER HALF-LINEAR DYNAMIC EQUATIONS ON TIME SCALES

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ABSTRACT. In this paper we establish some oscillation theorems for the second order half-linear dynamic equation

$$\left(r(t)(x^{\Delta}(t))^{\gamma}\right)^{\Delta} + p(t)x^{\gamma}(t) = 0, \quad t \in [a, b],$$

on time scales. Special cases of our results include some well-known oscillation results for second-order differential and half-linear differential equations. Our results are new for difference, generalized difference and q difference half-linear equations.

1. Introduction. The theory of time scales, which has recently received a lot of attention, was introduced by Stefan Hilger in his Ph.D. Thesis in 1988 in order to unify continuous and discrete analysis, see [**16**]. The theory of "dynamic equations" unifies the theories of differential equations and difference equations and it also extends these classical cases to cases "in between," e.g., to the so-called $q-$difference equations. A time scale $\mathbf{T}$ is an arbitrary closed subset of the reals, and the cases when this time scale is equal to the reals or to the integers represent the classical theories of differential and of difference equations. Many other interesting time scales exist, and they give rise to many applications, see [**5**]. Since Stefen Hilger formed the definition of derivatives and integrals on time scales, several authors have expounded on various aspects of this new theory, see the paper by Agarwal et al. [**1**] and the references cited therein. A book on the subject of time scale, i.e., measure chain, by Bohner and Peterson [**5**] summarizes and organizes much of time scale calculus, and in the next section, we recall some of the main tools used in the subsequent sections of this paper.

2000 AMS *Mathematics Subject Classification.* Primary 34K11, 34C10, 93C70.
Key words and phrases. Oscillation, second order half-linear dynamic equations, time scale.
Received by the editors on December 14, 2004.