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REGULAR SETS OF SAMPLING AND INTERPOLATION IN BERGMAN SPACES

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ABSTRACT. Let ρ denote the pseudohyperbolic metric in the unit disk $\mathbf{D}$ in the complex plane. We give examples of analytic functions g satisfying the condition $|g(z)| \simeq \rho(z,\Gamma)(1-|z|)^{-\alpha}$, $z \in \mathbf{D}$, in the case when Γ are A^p zero sets considered by Horowitz and Luecking. This helps to solve directly interpolating and sampling problems for these sequences.

1. Introduction. For $0 < p < \infty$, the Bergman space A^p is the set of functions analytic in the unit disk $\mathbf{D}$ with

$$\|f\|_p = \left(\int_{\mathbf{D}} |f(z)|^p dA(z)\right)^{1/p} < \infty,$$

where dA denotes the normalized Lebesgue area measure on $\mathbf{D}$.

A sequence $\{z_k\}$ of distinct points in $\mathbf{D}$ is an interpolation sequence for A^p, if the interpolation problem

$$f(z_k) = w_k, \quad k = 1, 2, \dots ,$$

has a solution $f \in A^p$ provided

$$\sum_{k=1}^{\infty}(1-|z_k|^2)^2\,|w_k|^p < \infty.$$

A sequence $\{z_k\}$ of distinct points in $\mathbf{D}$ is a sampling sequence for A^p if there exist positive constants K_1, K_2 such that

$$K_1\|f\|_p^p \le \sum_{k=1}^{\infty}(1-|z_k|^2)^2\,|f(z_k)|^p \le K_2\|f\|_p^p.$$

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