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THE GENERALIZED QUASILINEARIZATION FOR INTEGRO-DIFFERENTIAL EQUATIONS OF VOLTERRA TYPE ON TIME SCALES

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ABSTRACT. We apply the method of quasilinearization to integro-differential equations of Volterra type. It is shown that two monotone sequences converge quadratically to a unique solution of our problem.

1. Introduction. Throughout this paper, we denote by $\mathbf{T}$ any time scale (nonempty closed subset of real numbers $\mathbf{R}$). By $J = [0, T]$, we denote a subset of $\mathbf{T}$ such that $[0, T] = \{t \in \mathbf{T} : 0 \leq t \leq T\}$. By $C(J, \mathbf{R})$, we denote the set of continuous functions $u : J \to \mathbf{R}$.

In this paper, we investigate the following first order integro-differential equations of Volterra type on time scales

$$(1) \qquad \begin{cases} x^{\triangle}(t) = f\left(t, x(t), \displaystyle\int_0^t k(t,s)x(s)\triangle s\right) \equiv (\mathcal{F}x)(t) \quad t \in J, \\ x(0) = x_0 \in \mathbf{R}, \end{cases}$$

where $f \in C(J \times \mathbf{R} \times \mathbf{R}, \mathbf{R})$, $k \in C(J \times J, \mathbf{R})$.

The method of quasilinearization is a well-known technique for obtaining approximate solutions of nonlinear differential equations (for details, see for example [**7**] and references therein). There is a lot of application of this method to ordinary differential equations both with initial and boundary conditions. This technique can also be applied to corresponding problems on time scales (see, for example [**2, 3**]). In this paper, we apply the generalized quasilinearization method for integro-differential problems of Volterra type on time scales. The purpose of this paper is to exploit the recent ideas of this method applied to nonlinear differential equations (see, for example [**7**]). We investigate the

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