

## LIMITS OF VERTEX REPLACEMENT RULES

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ABSTRACT. In an earlier paper [10], J. Previte developed a framework for studying iterated replacements of certain vertices in a graph  $G$  by a finite replacement graph  $H$ . He showed that the normalized sequence of iterated graphs converges in the Gromov-Hausdorff metric (except for special cases). In this paper, we extend the framework in [10] to iterated vertex replacements where there are at least two replacement graphs and prove a convergence result. We also give examples of vertex replacement rules that yield convergent sequences of graphs.

**1. Introduction.** The notion of vertex replacement rules was motivated by studying the horospheres of the geodesic flow on a two-dimensional singular space  $X$  of nonpositive curvature, see [1].

For two-dimensional singular spaces of nonpositive curvature, the horospheres of  $X$  are graphs. The work in this paper is also related to a class of iterative systems, introduced by Aristid Lindenmayer, see [12, 13], which is used to model the growth of plants and simple multicellular organisms. Lindenmayer theorized that the development of a complex object, such as a plant, must be governed by a relatively simple set of production rules. His approach created a new branch of biomathematics. Lindenmayer systems were later used in the areas of data and image compression. Since the systems introduced in this paper are more natural and geometric, they promise applications in the same fields that Lindenmayer impacted.

A vertex replacement rule  $\mathcal{R}$  is a rule for substituting copies of finite graphs, called replacement graphs, for certain vertices in a given graph  $G$ . The result is a new graph  $\mathcal{R}(G)$ . Iterating  $\mathcal{R}$  produces a sequence of graphs  $\mathcal{R}^n(G)$ . By letting  $(\mathcal{R}^n(G), 1)$  be the metric space  $\mathcal{R}^n(G)$  normalized to have diameter 1, the sequence of the normalized graphs can be studied.

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