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CURVES ARISING FROM ENDOMORPHISM RINGS OF KRONECKER MODULES

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ABSTRACT. In this note we prove that the endomorphism ring of a Kronecker module attached to a power series $\alpha \in k[[X]]$ is minimally generated by three generators, unless its degree d is less than 3. We prove this via the theory of algebraic curves, by proving that none of the affine curves arising from these endomorphism rings are planar for $d \geq 3$, but can always be embedded in $\mathbf{A}^3$.

1. Introduction. In their paper [**4**], Okoh and Zorzitto describe a family of k-algebras A_α attached to Kronecker modules P_α, where α is a power series with coefficients in an algebraically closed field k. The algebra A_α has a degree $d = \deg(A_\alpha)$, and Okoh and Zorzitto prove that if $d \leq 2$, then the algebra A_α is minimally generated as a k-algebra by d elements. They then ask how many elements will minimally generate an algebra A_α of arbitrary degree. The purpose of this note is to show that the answer is $\min\{3, d\}$.

Okoh and Zorzitto prove their result for $d = 1$ and $d = 2$ using the theory of algebraic curves. In this paper, we will take this philosophy further and derive general results using more sophisticated geometric tools. Precisely, in Section 2, we recall the relevant definitions and notation from [**4**]. To keep the paper to a manageable length, we describe only the definitions and properties we use in this paper; for a full description of the Kronecker modules and associated constructions, we refer the reader to [**4**]. In Section 3, we give a careful description of the geometry of the curves under consideration, and in Section 4 we prove the main result. Section 4 in particular contains some quite technical algebraic geometry; we refer the reader to [**2**] and [**1**] for definitions and explanations of any unfamiliar geometric or algebraic terms.

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