

## OPERATOR ALGEBRAS AND MAULDIN-WILLIAMS GRAPHS

MARIUS IONESCU

**ABSTRACT.** We describe a method for associating a  $C^*$ -correspondence to a Mauldin-Williams graph and show that the Cuntz-Pimsner algebra of this  $C^*$ -correspondence is isomorphic to the  $C^*$ -algebra of the underlying graph. In addition, we analyze certain ideals of these  $C^*$ -algebras.

We also investigate Mauldin-Williams graphs and fractal  $C^*$ -algebras in the context of the Rieffel metric. This generalizes the work of Pinzari, Watatani and Yonetani. Our main result here is a “no go” theorem showing that such algebras must come from the commutative setting.

**1. Introduction.** In recent years many classes of  $C^*$ -algebras have been shown to fit into the Pimsner construction of what are known now as Cuntz-Pimsner algebras, see [20, 22]. This construction is based on a so-called  $C^*$ -correspondence over a  $C^*$ -algebra. For example, a natural  $C^*$ -correspondence can be associated with a graph  $G$ , see [10], [11, Example 1.5]. The Cuntz-Pimsner algebra of this  $C^*$ -correspondence is isomorphic to the graph  $C^*$ -algebra  $C^*(G)$  as defined in [16]. Another example is the  $C^*$ -correspondence associated with a local homeomorphism on a compact metric space studied by Deaconu in [6], and the  $C^*$ -correspondence associated with a local homeomorphism on a locally compact space studied by Deaconu, Kumjian, and Muhly in [7]. They showed that the Cuntz-Pimsner algebra is isomorphic to the groupoid  $C^*$ -algebra associated with a local homeomorphism in [5, 7, 26].

By a (directed) *graph* we mean a system  $G = (V, E, r, s)$  where  $V$  and  $E$  are finite sets, called the sets of *vertices* and *edges*, respectively, of the graph, and where  $r$  and  $s$  are maps from  $E$  to  $V$ , called the *range* and *source* maps, respectively. Thus,  $s(e)$  is the source of an edge  $e$  and  $r(e)$  is its range. A *Mauldin-Williams graph* is a graph  $G$  together with a collection of compact metric spaces, one for each

---

AMS *Mathematics subject classification.* Primary 26A18, 37A55, 37B10, 37E25, 46L08, 46L55, 46L89.

Received by the editors on Jan. 28, 2004, and in revised form on Nov. 29, 2004.

Copyright ©2007 Rocky Mountain Mathematics Consortium