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SUBGROUPS OF PURE BRAID GROUPS GENERATED BY POWERS OF DEHN TWISTS

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ABSTRACT. Let B_n be the group of braids on n strings, and let P_n be the corresponding pure braid group. In this paper we consider subgroups of B_n generated by powers of Dehn twists. For example, let $A_{12}, A_{13}, \dots, A_{n-1,n}$ be the standard Dehn twist generators for P_n and consider subgroups of the form $\langle A_{ij}^{\varepsilon_{ij}} \rangle$; we give conditions guaranteeing that such a subgroup has finite index in P_n. We then consider subgroups obtained by adding in powers of other Dehn twists. In the cases considered the finite index property is characterized in terms of certain inequalities.

1. Introduction. The braid group B_n has the presentation

$$\left\langle \sigma_1, \dots, \sigma_{n-1} \,\middle|\, \begin{matrix} \sigma_i\sigma_{i+1}\sigma_i = \sigma_{i+1}\sigma_i\sigma_{i+1}, \ 1 \le i < n-1; \\ \sigma_i\sigma_j = \sigma_j\sigma_i, \ |i-j| > 1 \end{matrix} \right\rangle.$$

This makes it clear that there is an epimorphism $B_n \to S_n, \sigma_i \mapsto (i, i+1)$. The kernel of this map is P_n, the *pure braid group* of index $n!$. It is well known [**1**] that P_n is generated by elements $A_{ij}, \ 1 \le i < j \le n$, where

$$A_{ij} = \sigma_i^{-1} \cdots \sigma_{j-2}^{-1} \, \sigma_{j-1}^{-1} \, \sigma_j^2 \, \sigma_{j-1} \, \sigma_{j-2} \cdots \sigma_i.$$

A presentation for P_n with these generators is indicated in [**1, 5, 7**]. It thus seems natural to investigate subgroups of the form

$$H = \langle A_{ij}^{\varepsilon_{ij}} \mid 1 \le i < j \le n \rangle, \tag{1.1}$$

which we call A_{ij} *subgroups*. Other relevant results on properties of Dehn twists and groups generated by Dehn twists can be found in [**4, 8**].

For H as in (1.1) the criterion for $[P_n : H]$ to be finite is given in

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