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ON PRIME SUBMODULES

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Throughout this paper R will denote a commutative ring with identity and M a unital module. Several authors have extended the notion of prime ideal to modules, see, for example [**1, 2**]. In this paper, we continue these investigations.

A proper submodule N of M is prime if for any $r \in R$ and $m \in M$ such that $rm \in N$, either $rM \subseteq N$ or $m \in N$. It is easy to show that if N is a prime submodule of M then the annihilator P of the module M/N is a prime ideal of R. Also it is not difficult to see that N is a prime submodule of M if and only if $(N : K) = (N : M)$ for all submodules K of M properly containing N.

It is well known that a submodule N of M is prime if and only if $P = (N : M)$ is a prime ideal of R and the (R/P)-module M/N is fully faithful. For a prime ideal P of R, McCasland and Smith [**8**] defined the set $M(P)$ and asked the question: When does $M = M(P)$? In this paper we give an answer to this question and also describe the interrelation between the attached primes and prime submodules of an Artinian R-module.

Let N be a proper submodule of an R-module M. The radical of N in M, denoted by $\mathrm{rad}_M N$, is defined to be the intersection of all prime submodules of M containing N. Should there be no prime submodule of M containing N, then we put $\mathrm{rad}_M N = M$. On the other hand, $\mathrm{rad}\, R$ denotes the intersection of all prime ideals of R. Let I be an ideal of R. Then it is well known that $\sqrt{I} = \{r \in R : r^n \in I \text{ for some } n \in \mathbf{N}\}$. The envelope submodule $RE_M(N)$ of N in M is a submodule of M generated by the set $E_M(N) = \{rm : r \in R \text{ and } m \in M \text{ such that } r^n m \in N \text{ for some } n \in \mathbf{N}\}$.

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