ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 33, Number 4, Winter 2003

ON CONTINUOUS SOLUTIONS OF A FUNCTIONAL EQUATION OF ITERATIVE TYPE

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ABSTRACT. Properties of continuous solutions of the functional equation $\sum_{i=1}^{n} \lambda_i f^{2i-1}(x) = F(x)$ are discussed. Under some conditions we prove the existence, uniqueness and stability of the continuous solutions of the equation.

1. Introduction. The iterative equation

$$f^n(x) = F(x), \tag{1.1}$$

is an important form of functional equations, where $f : I = [a, b] \to I$ is an unknown function, f^n denotes the n-th iterate of f. Abel [**1**], Bödewadt [**2**], Dubbey [**4**], Fort [**6**], Kuczma [**7, 8**] and others established the existence of solutions for equation (1.1). It is well known that equation (1.1) has a continuous solution for any n if F is a strictly increasing continuous function and equation (1.1) has no continuous solutions for even n if F is a strictly decreasing continuous function. Recently, a few elegant results for equation

$$\sum_{i=1}^{n} \lambda_i f^i(x) = F(x) \tag{1.2}$$

have been obtained in [**3**] and [**9–12**]. In particular, Zhang [**10,11**] discussed the existence, uniqueness and stability of continuous solutions of equation (1.2), where F is a strictly increasing continuous function in $[a, b]$ and has fixed points a, b.

1991 AMS *Mathematics Subject Classification.* 39B12, 39B20.
Key words and phrases. Functional equation, Ascoli-Arzela lemma, continuous solution, existence, uniqueness, stability, Schauder's fixed point theorem, Edelstein's fixed point theorem.
The second author was supported by Korea Research Foundation grant (KRF-2001-015-DP0025).
Received by the editors on October 31, 2000, and in revised form on August 23, 2001.