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SOLUTION TO TWO PROBLEMS IN INVERSE INTERPOLATION

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ABSTRACT. Answering two problems raised by A.L. Horwitz and L.A. Rubel, we construct analytic functions f such that $L(L(f))$ is the set of all polynomials (here $L(f)$ denotes the set of all Lagrange interpolants of f on $[0,1]$).

Let $L(f)$ denote the set of all Lagrange interpolants based on knots in $[0,1]$ of the function f defined on $[0,1]$. A.L. Horwitz and L.A. Rubel [**1**] proved that if f and g are analytic on $[0,1]$ and $L(f)=L(g)$, then $f=g$; on the other hand, they constructed a large class of C^∞ - functions f for which $L(L(F))$ is the set of all polynomials. They asked

PROBLEM 1. *Is there a function f analytic on $[0,1]$ such that $L(L(f))$ is the set of all polynomials*?

and

PROBLEM 2. *If f and g are analytic on $[0,1]$ and $L(L(f))=L(L(g))$, then must $f=g$*?

In this paper we show that there are many analytic functions of the form

$$f(z)=\int_{\mathbf{R}}\frac{d\mu(t)}{1+tz},\quad z\in[0,1],$$

where μ is a finite signed measure, for which $L(L(f))$ is the set of all polynomials. Hence, the answer is positive for Problem 1 and it is negative for Problem 2.

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