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A NOTE ON COMPACTNESS IN L-FUZZY PRETOPOLOGICAL SPACES

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ABSTRACT. The main task of this paper is to introduce and study the concept of countable compactness, Lindelöf, almost compactness and near compactness in L-fuzzy pretopological spaces. Also, the images of such spaces are investigated. Finally, some examples of the above spaces are given.

1. Introduction. Throughout this paper, the symbol L will denote a complete lattice, with a smallest element 0 and a largest element 1, that is equipped with an order-reversing involution; for such a lattice the DeMorgan laws hold for arbitrarily indexed suprema and infima.

Let X be a nonempty set and $L^X = \{A : X \to L\}$. The elements of L^X are called *L-fuzzy subsets* of X[**4**]. If $A \in L^X$, then $A^c = 1 - A$. We denote 0_X and 1_X for the functions on X identically equal to 0 and 1 respectively. If $f : X \to Y$ and $B \in L^Y$, then $f^{-1}(B) = B \circ f$. One proves that $f^{-1} \vee_{j\in J} B_j = \vee_{j\in J} f^{-1}(B_j)$ and $\wedge_{j\in J} B_j) = \wedge_{j\in J} f^{-1}(B_j)$. If $f : X \to Y$ and $A \in L^X$, then $f(A) : Y \to L$ is defined by setting $f(A)(y) = 0$ if $f^{-1}(y) = \phi$ and $f(A)(y) = \vee_{y=f(x)} A(x)$ otherwise. One proves that $f(f^{-1}(A)) \leq A$, and if f is surjective, then $f(f^{-1}(A)) = A$. Yet, $f^{-1}(f(A)) \geq A, f(\wedge_{j\in J} A_j) \leq \wedge_{j\in J} f(A_j)$ and $f(\vee_{j\in J} A_j) = \vee_{j\in J} f(A_j)$. A *fuzzy point* P in X is an L-fuzzy set in X defined by: $P(x) = t$ for $x = x_0$ and $P(x) = 0$ otherwise. The point x_0 is the support of P, and $0 < t < 1$. For a fuzzy point P in X and $A \in L^X, P \in A$ if $P(x_0) < A(x_0)$ [**9**]. A collection $\{A_j\}_{j\in J}$, where $A_j \in L^X, \forall A_j \in L^X \forall_j \in J$, is a cover of X if and only if $\vee_{j\in J} A_j = 1_X$.

An L-fuzzy pretopology [**2**] on X is a function $a : L^X \to L^X$ which satisfies:

(P1). $a(\phi) = \phi$,

(P2). $a(A) \geq A$, for every $A \in L^X$.

The pair (X, a) is said to be an L-fuzzy pretopological space (for short, L-fps). We will consider the following particular L-fps:

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