ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 20, Number 1, Winter 1990

WHEN DO TWO GROUPS ALWAYS HAVE ISOMORPHIC EXTENSION GROUPS?

H. PAT GOETERS

What is the relationship between abelian groups A and C if $\mathrm{Ext}(A,B) \cong \mathrm{Ext}(C,B)$ for all abelian groups B? (problem 43 in [**5**]). We will address this question, restricting our attention to torsion-free abelian groups A, B and C of finite rank.

Call A and C *related* if $\mathrm{Ext}(A,B) \cong \mathrm{Ext}(C,B)$ for all B. We give a characterization of this relation in §1 and use it to show

THEOREM. *Assume that one of the following hold*: (a) $\mathrm{rank}\, A = 2$; (b) *A has a semi-prime endomorphism ring*; *or* (c) *A is almost completely decomposable. Write* $A = D' \oplus F' \oplus G$ *with* F' *free*, D' *divisible and* G *reduced with* $\mathrm{Hom}(G, \mathbf{Z}) = 0$.

Then C *is related to* A *if and only if* $C = D \oplus F \oplus R$ *with* F *free*; D *is divisible and zero if* $\mathrm{OT}(A) \neq$ type $\mathbf{Q}$ *and nonzero if* $\mathrm{OT}(G) \neq$ type $\mathbf{Q}$ *and* $D' \neq 0$; *and* R *quasi-isomorphic to* G.

Here $\mathbf{Z}$ is the ring of integers and $\mathbf{Q}$ the field of rationals, p will denote a prime of $\mathbf{Z}$. As usual, the p-rank of A, $r_p(A) = \dim A/pA$. We show the

COROLLARY. *Assume that one of the following hold*: (a) $\mathrm{rank}\, A = 2$; (b) *A has a semi-prime endomorphism ring*; *or* (c) *A is almost completely decomposable. Then C is quasi-isomorphic to A if and only if* (i) $r_p(C) = r_p(A)$ *for all* p; (ii) $r_p(\mathrm{Hom}(C,B)) = r_p(\mathrm{Hom}(A,B))$ *for all* p *and groups* B *with* $\mathrm{rank}\, B \leq \mathrm{rank}\, A$; (iii) $\mathrm{OT}(C) = \mathrm{OT}(A)$; *and* (iv) $\mathrm{rank}\, C = \mathrm{rank}\, A$.

The notation, if undefined, appears in [**1**], and the basic ideas from [**1**] are assumed. However a few facts about the outer type of $A, \mathrm{OT}(A) =$

Received by the editors on August 7, 1986 and in revised form on April 7, 1987.