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THE ROTATIONS OF $\ell(\phi_{\mathbf{n}})$

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ABSTRACT. A characterization of the linear rotations of a general class of metric linear spaces is given. Sufficient conditions are given for all the rotations of these spaces to be linear.

1. Introduction. Let (p_n) be a sequence of real numbers with $0 < p_n \leq 1$. The linear space $\ell(p)$ is defined [**2**] to be the collection of all real (or complex) sequences (x_n) for which $\sum_{n=1}^{\infty} |x_n|^{p_n}$ is finite. It is a complete metric linear space with the metric given by

$$d(x,y) = \sum_{n=1}^{\infty} |x_n - y_n|^{p_n}. \tag{1.1}$$

A surjective mapping T (not necessarily linear) of a metric linear space (X, d) is a rotation [**1, 2**] if $T(0) = 0$ and $d(Tx, Ty) = d(x, y)$. In [**2**] Maddox asks for a description of the rotations of $\ell(p)$. This question provided the motivation for the present paper.

In this paper we describe a broad class of metric linear spaces, denoted by $\ell(\phi_n)$, which include $\ell(p)$. We characterize, in terms of the action on the space, the linear rotations of $\ell(\phi_n)$. In general it is not known if a rotation of an arbitrary metric linear space is a linear transformation. In fact, in spaces over the complex field this need not be the case. However, for metric linear spaces over the real field, sufficient conditions for rotations to be linear are known [**2**]. In §2 we indicate which of the $\ell(\phi_n)$ spaces satisfies these conditions. Finally, we note that our results answer the question of Maddox except for $\inf p_n = 0$.

2. The spaces $\ell(\phi_{\mathbf{n}})$. Let (ϕ_n) be a sequence of continuous real valued functions defined on $[0, \infty)$ such that $\phi_n(0) = 0, \phi_n(1) = 1, \phi_n(\cdot)$ is increasing, and $\phi_n{}'(\cdot)$ is decreasing.

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