ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 20, Number 1, Winter 1990

COUNTING FINITE SUBSETS OF AN IMMUNE SET

J.C.E. DEKKER

ABSTRACT. Let P be a recursive property of finite sets (of nonnegative integers) and ν an immune set of RET (i.e., recursive equivalence type) N. Consider the question, "How many finite subsets of ν have property P?" We shall answer this question if P has the additional property that $P(\alpha)$ if and only if $P(\beta)$, for every two finite sets α and β of the same cardinality.

1. Preliminaries. We use the word *number* for nonnegative integer, *set* for collection of numbers and *class* for collection of sets. The set of all numbers and the empty set are denoted by ε and o respectively. V stands for the class of all sets, Q for the class of all finite sets, $\subset$ for inclusion and $\subset_+$ for proper inclusion. If f is a function, δf and ρf denote its domain and range respectively. Also, f_n means the same as $f(n)$. The cardinality of a collection Γ is denoted by $\mathrm{card}(\Gamma)$ or $\mathrm{card}\,\Gamma$. The reader is assumed to be familiar with the basic properties of the collection Λ of all isols. For a survey of basic results see §1 of [**2**]; for a detailed exposition, see [**4**] or [**5**]. We write $\alpha \sim \beta$ for α is equivalent to β, i.e., $\mathrm{card}\,\alpha = \mathrm{card}\,\beta$ and $\alpha \simeq \beta$ for α is recursively equivalent to β, i.e., $\mathrm{Req}(\alpha) = \mathrm{Req}(\beta)$. Some properties of combinatorial operators will be used; these are discussed in [**1**], [**4**] and [**5**]. The class Q of all finite sets can be effectively generated without repetitions in an infinite sequence. We shall use a particular sequence of this type, the so-called *canonical enumeration* $\langle \rho_n \rangle$; see [**2**, p. 277]. For each finite set σ there is exactly one number i such that $\sigma = \rho_i$; this number is called the *canonical index* of σ and is denoted by $\mathrm{can}(\sigma)$ or $\mathrm{can}\,\sigma$. If $S \subset Q$ we write $\mathrm{can}\,S$ for $\{\mathrm{can}\,\sigma | \sigma \in S\}$. The function $r_n = \mathrm{card}\,\rho_n$ is recursive. For $n, k \in \varepsilon$ and $\alpha \in V$,

$$\nu_n = \{x \in \varepsilon \,|\, x < n\}, \qquad 2^\alpha = \{x \in \varepsilon \,|\, \rho_x \subset \alpha\},$$
$$[\alpha; k] = \{x \in 2^\alpha \,|\, r_x = k\}, \qquad \int [n; k] = \mathrm{card}\,[\nu_n; k].$$

AMS (MOS) *Subject Classification* (1980). 03D50.
Received by the editors on February 16, 1987.