ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 20, Number 1, Winter 1990

ORDER CONTINUOUS BOREL LIFTINGS

DAVID C. CAROTHERS

Introduction. The lifting theorem of A. and C. Ionescu-Tulcea [**3**] can be stated as follows: Every bounded linear operator $T : L^\infty_\mu \to L^\infty_\mu$ has a lifting $\hat{T}$, taking values in M, the space of bounded μ-measurable functions. In other words, $P_\mu \circ \hat{T} = T$, where P_μ is the natural projection of M onto equivalence classes in L^∞_μ.

It is not known whether M may be replaced by the space of Borel functions in the Ionescu-Tulcea theorem. In this paper we study order continuous operators on L^∞_μ and characterize those which have an order continuous lifting $\hat{T}$ which takes values in the Borel functions.

Let X be a compact Hausdorff space and let μ be a positive bounded Baire measure on X. $C(X)$, or C, is the space of continuous functions on X, with first and second normal duals C' and C''. μ may be identified with a positive element of C'. C, C', and C'' are Riesz spaces, or vector lattices, and C may be embedded in C'' in a natural way.

This paper will deal with C, C', and C'' along with various subspaces which are order isomorphic with L^1_μ, L^∞_μ, and the space of Borel functions. For a thorough study of C'' and definitions not included here, see [**4**]. For more information on Riesz spaces, see Schaefer [**6**] or Luxemburg-Zaanen [**5**].

C' may be written as the order direct sum of C'_a, the "atomic" measures (those generated by X as a subset of C') and C'_d, the "diffuse" measures (those order disjoint from C'_a). This yields a corresponding decomposition $C'' = C''_a \oplus C''_d$, where $C''_a = (C'^{\perp}_a)^d$. $C^u(C^l)$ consists of those elements of C'' which are infima (suprema) of subsets of C. $s(X) = C^u - C^u = C^l - C^l$ is the linear subspace generated by C^l or C^u. The σ-order closure of $s(X)$ will be denoted by Bo. Bo is order isomorphic with its projection Bo_a onto C''_a (and thus is determined by its values on $X \subset C'$).

Received by the editors on October 8, 1986 and in revised form on April 3, 1987.