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APPROXIMATION BY SEMI-FREDHOLM OPERATORS WITH FIXED NULLITY

RICHARD BOULDIN

1. Introduction. Let H be a fixed complex separable Hilbert space. For any (bounded linear) operator T on H, we define the nullity and deficiency, denoted $\operatorname{nul} T$ and $\operatorname{def} T$, to be the dimensions of the kernels of T and T^*, respectively. Of course, the index of T, denoted $\operatorname{ind} T$, is defined to be $(\operatorname{nul} T - \operatorname{def} T)$, with $\infty - \infty$ understood to be 0. We denote the operator norm of T by $\|T\|$ and the spectrum by $\sigma(T)$.

In [**2**] the distance from an arbitrary operator T to the set of invertible operators (and to the Fredholm operators) was determined. This provided a refinement of the classical result in [**5**] that describes the closure of the invertible operators. Subsequently Theorem 12.2 in [**1**] elaborated on [**2**] by showing that the formula given there was actually the distance from an arbitrary operator T to each set of semi-Fredholm operators with an index different from that of T. [**1**] went on to show that the preceding theorem plays a significant role in similarity theory.

In [**7**] the original methods of [**2**] are used to modify Theorem 12.2 to obtain the distance from T to the right invertible operators with a fixed nullity. All of the preceding results and some new methods were used in [**3**] to find the distance from T to the (unrestricted) set of operators with a fixed nullity; the formula obtained in [**3**] is a striking contrast to previously obtained formulas. In this note we determine the distance from T to a natural set which contains the right invertible operators with nullity equal to n and is contained in the set of operators with nullity equal to n. The results have some resemblance to those in [**3**] and some to those in [**7**].

2. Preliminaries. This section contains results that will be used frequently in the subsequent section. These results will be used sometimes without citation.

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