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ON SUMS OF UNISERIAL MODULES

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Let R be any ring. Following [**2**] a module M_R is called a TAG-module if it satisfies the following two conditions.

(I) Every finitely generated submodule of a homomorphic image of M is a direct sum of uniserial modules.

(II) Given any two uniserial submodules U and V of a homomorphic image of M, for any submodule W of U, any homomorphism $f : W \to V$ can be extended to a homomorphism $g : U \to V$ provided the composition length $\mathrm{d}(U/W) \leq \mathrm{d}(V/f(W))$.

A module M_R satisfying condition (I) is called a QTAG-module [**9**]. Through a number of papers it has been seen that the structure theory of these modules is similar to that of torsion abelian groups and that these modules occur over any ring. In this paper, in addition to further developing their structure theory, we give some applications of these modules to ring theory. In §2, Proposition 2.3 and Theorem 2.5 give some new characterizations of TAG-modules and QTAG-modules respectively. In §3 we determine when the class of QTAG modules over a ring R is closed under direct sums and use these results to give some characterizations of generalized uniserial rings (Theorem 3.5). Even if the class of QTAG-modules over a ring is closed under direct sums, it need not be closed under extensions. In §4 we determine when, over a commutative ring R, the class of TAG-modules is closed under extensions; these rings are precisely those which do not admit any homomorphic image which is a special ring in the sense defined by Shores [**6**, Theorem 4.9].

1. Preliminaries. All the rings considered here are with unity, and the modules are unital right modules unless otherwise stated. For any module M with finite composition length, $\mathrm{d}(M)$ denotes its (composition) length. For any module M_R, $\mathrm{J}(M)$ and $\mathrm{E}_R(M)$ (or

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