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THE SPATIAL FORM OF ANTIAUTOMORPHISMS OF VON NEUMANN ALGEBRAS

ERLING STØRMER

1. Introduction. There are three problems which have been studied concerning antiautomorphisms of von Neumann algebras; the existence problem, the conjugacy problem, and their description. The latter problem includes whether they are spatial of a particular form, i.e., of the form $x \to w^* x^* w$ with w a conjugate linear isometry of a prescribed type. In the present paper we shall study the spatial problem, with main emphasis on antiautomorphisms α leaving the center elementwise fixed, called *central* in the sequel, and with α an *involution*, i.e., $\alpha^2 = 1$. This problem with variations has previously been studied in [**2, 6**]. E.g., it was shown in [**6**] that a central involution α is automatically spatial with w^2 a selfadjoint unitary operator in the center of the von Neumann algebra.

It turns out that the general problem of whether a central antiautomorphism is spatial has a solution similar to that of automorphisms, with proof also quite similar. We include these results for the sake of completeness. The main new ingredient in the paper is that if α is a central involution of the von Neumann algebra M then α is necessarily of the form $\alpha(x) = Jx^*J$ with J a conjugation, unless the commutant M' of M has a direct summand of type I_n with n odd. In the latter case it may happen that α can only be written in the form $\alpha(x) = -jx^*j$ with $j^2 = -1$.

2. The results. Recall that two projections e and f in a von Neumann algebra M acting on a Hilbert space H are said to be equivalent, written $e \sim f(\text{mod}\, M)$, or just $e \sim f$ if there is a partial isometry $v \in M$ such that $v^*v = e$, $vv^* = f$. e is said to be cyclic, written $e = [M'\xi]$ if there is a vector $\xi \in H$ such that e is the projection onto the space spanned by vectors of the form $x'\xi,\ x' \in M'$. If w is a conjugate linear operator we denote by w^* its adjoint, viz, $(w^*\xi, \eta) = (w\eta, \xi)$. We denote by ω_ξ the positive functional $\omega_\xi(x) = (x\xi, \xi)$ on M.

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