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AF SUBALGEBRAS OF CERTAIN CROSSED PRODUCTS

YIU TUNG POON

ABSTRACT. Let (X,T) be a dynamical system with X zero dimensional. Each closed subset Y of X gives rise to a subalgebra A_Y of the crossed product C^*-algebra $C(X)\times_T \mathbf{Z}$. We give a necessary and sufficient condition on Y for A_Y to be an AF algebra. Suppose Y_1 and Y_2 are two clopen subsets satisfying the condition. We show that Y_1 and Y_2 are homeomorphic as topological spaces if and only if the AF algebras A_{Y_1} and A_{Y_2} are stably isomorphic. Finally, we show that, if the non-periodic points are dense in X and Y is a minimal subset satisfying the condition, then A_Y is a maximal AF subalgebra among the regular subalgebras of $C(X)\times_T \mathbf{Z}$.

1. Introduction. Given a compact space X, $C(X)$ will denote the C^*-algebra of complex continuous functions on X. A compact metrizable space X is said to be zero dimensional if the topology on X has a basis consisting of sets which are both closed and open (clopen). In this note we study systems (X,T) where X is a zero dimensional space and T is a homeomorphism on X. Given such a system,we have an action of the integers $\mathbf{Z}$ on $C(X)$. This gives a crossed product algebra $C(X)\times_T \mathbf{Z}$ (see Pedersen [**5**]) which is a C^*-algebra generated by $C(X)$ and a unitary U satisfying $UfU^* = f\circ T^{-1}$ for $f\in C(X)$. In [**7**], we show that the order structure on $K_0(C(X)\times_T \mathbf{Z})$ is useful in the study of classification problems of such systems and the crossed product algebras. (We will use Blackadar [**1**] and Effros [**3**] for our reference on K-theory). A system (X,T) is said to be minimal if X contains no non-empty T-invariant proper closed subsets. In recent works [**9, 10**], Putnam proved, among other results, that if X does not have isolated points and the system (X,T) is minimal, then, for every closed subset Y, the C^*-subalgebra of $C(X)\times_T \mathbf{Z}$ generated by $C(X)$ and $\{Uf : f\in C(X), f(y)=0 \text{ for all } y\in Y\}$ is an AF algebra [**9, 10**], i.e., A_Y is the closure of an increasing sequence of finite dimensional subalgebras. This result is crucial in his study of AF-subalgebras of $C(X)\times_T \mathbf{Z}$ [**10**] and the order structure of $K_0(C(X)\times_T \mathbf{Z})$ [**9**]. In §2, given any (X,T) (not necessarily minimal) and a closed subset Y, we prove that A_Y is an AF algebra if and only if, for every clopen subset