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INVERTIBILITY AND TOPOLOGICAL STABLE RANK FOR SEMI-CROSSED PRODUCT ALGEBRAS

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0. Introduction. The computation of invariants for C^*-algebras such as the K-groups and topological stable rank, which has attracted so much attention in recent years, can be fruitful for nonselfadjoint operator algebras as well, though relatively little has been done in that direction. In the present discussion we will compute these invariants for a special class of nonselfadjoint, norm closed operator algebras, which we have called semi-crossed products [**4**]. These algebras include the subalgebras of C^* crossed products of $C(X)$ by a homeomorphism of X which are generated by the nonnegative powers of the homeomorphism.

Denote the semi-crossed product of $C(X)$ with respect to a homeomorhpism φ by $\mathbf{Z}^+ \times_\varphi C(X)$. As the invertible elements are never dense, the topological stable rank is greater than one, and we show it is in fact equal to two in case X is zero or one dimensional. (In particular, we show that the right and left stable ranks coincide, which is not automatic since the algebra is not involutive.) On the other hand, the K-theory for these algebras is "contractible" to K-theory of $C(X)$.

As the class of algebras we will be discussing is somewhat more general than the class of those algebras which arise naturally as subalgebras of C^*-crossed products, we will need some preliminaries. By a dynamical system we mean a pair (X, φ) where X is a compact Hausdorff space and $\varphi : X \to X$ is a continuous surjection. Denote by $K(\mathbf{Z}^+, C(X))$ the algebra over $\mathbf{C}$ which is the free product of $C(X)$ with a single operator, U together with the relations $fU = Uf \circ \varphi, f \in C(X)$. A typical $F \in K(\mathbf{Z}^+, C(X))$ thus has the form of a polynomial, $F = f_0 + Uf_1 + \cdots + U^n f_n$ where $n \in \mathbf{Z}^+, f_0, \dots, f_n \in C(X)$. For $x \in X$ define a representation of $K(\mathbf{Z}^+, C(X))$ on ℓ_2^+ by means of

$$\Pi_x(U)(\xi_0, \xi_1, \xi_2, \dots) = (0, \xi_0, \xi_1, \xi_2, \dots)$$

(the unilateral shift), and

$$\Pi_x(f)(\xi_0, \xi_1, \xi_2, \dots) = (f(x)\xi_0, f \circ \varphi(x)\xi_1, f \circ \varphi^2(x)\xi_2, \dots), \quad f \in C(X).$$

For $F = \sum_0^n U^k f_k$, set $\Pi_x(F) = \sum_0^n \Pi_x(U)^k \Pi_x(f_k)$. One checks that this defines a representation of $K(\mathbf{Z}^+, C(X))$. Obtain a norm on