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# A BI-MEASURABLE TRANSFORMATION GENERATED BY A NON-MEASURE PRESERVING TRANSFORMATION

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**0. Introduction.** In [**2**] V.A. Rohlin constructs an automorphism $S$ on a probability space associated with a measure preserving transformation $T$ on a given probability space. Questions concerning ergodicity, etc. about $T$ may be examined in terms of $S$. In this note we show that with some extra restrictions on $T$ (notably that $T$ takes measurable sets to measurable sets), a similar construction of a bi-measurable bijection is possible, without having $T$ being measure preserving. Moreover the state space need only be $\sigma$-finite. It is then shown that the composition operators on the various $L^p$ spaces constructed in terms of $S$ are extensions of the corresponding operators defined in terms of $T$. Moreover the extension does not increase the operator norm. The state space for the constructed measure is the inverse limit space as given in [**1**, Chapter 10]. Since the setting below is considerably different, several details of its construction and properties are included. It is noted that if $T$ is measure preserving, this procedure reduces to the standard case.

**1.** Let $(X, \sum, m)$ be a $\sigma$-finite measure space and let $T$ be a mapping of $X$ onto $X$ such that $T^{-1}\sum \subset \sum$ and $T\sum \subset \sum$. We assume that $m \circ T$ and $m \circ T^{-1}$ are mutually absolutely continuous with respect to $m$, where $m \circ T$ is viewed only as a function. Define

$$Y = \{y = \langle y_i \rangle | \text{ for each } i \geq 0, y_i \in X \text{ and } Ty_{i+1} = y_i\}.$$

Since $TX = X$ it follows from the countable axiom of choice that, for each $x \in X$ and each $n \geq 0$, there is a point $y$ in $Y$ with $y_n = x$. For each $A \in \sum$ and $n \geq 0$ let

$$(A)_n = \{y \in Y | y_n \in A\}$$

and define

$$F = \{(A)_n | A \in \sum, n \geq 0\}.$$

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