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LIFTING HANKEL OPERATORS FROM THE HARDY SPACE TO THE BERGMAN SPACE

PRATIBHA G. GHATAGE

This article is about lifting bounded Hankel operators on the Hardy space of the disk to (bounded) Bergman-Hankel operators on the Bergman space of the disk. Though Hankel operators on the Bergman space have been studied extensively [**2, 4**], the natural map that produces this lifting generates some questions about its range, leading to interesting function-theory.

Let $\mathcal{L}_a^2$ be the Bergman space of analytic functions on the open unit disk $\mathbf{D}$ that belong to $\mathcal{L}^2(\mathbf{D})$ with respect to the normalized area measure. Let B denote the Bloch space, consisting of functions f analytic on the disk which satisfy the condition $\sup_{|z|<1}(1-|z|)|f'(z)| < \infty$. We write $||f||_B = \sup_{|z|<1}(1-|z|^2)|f'(z)|$. The little Bloch space B_0 is the subspace of the Bloch space consisting of functions for which $\lim_{|z|\to 1}(1-|z|)|f'(z)| = 0$. It is well-known that B_0 is the closure of polynomials in the Bloch norm [**1**]. A natural way to define a Hankel operator is as follows: If $\phi \in \mathcal{L}_a^2(\mathbf{D})$, let $S_\phi h = PJ(\phi h)$, where J is the self-adjoint unitary operator given by $(JF)(z) = f(\overline{z})$ and P is the orthogonal projection of $\mathcal{L}^2(\mathbf{D})$ onto $\mathcal{L}_a^2(\mathbf{D})$. For $\overline{\phi} \in \mathcal{L}_a^2(\mathbf{D})$ it is well-known that S_ϕ is a bounded operator on $\mathcal{L}_a^2$ if and only if $\overline{\phi}$ belongs to the Bloch space, in which case $||\overline{\phi}||_B \approx ||S_\phi||$ [**4**]. If $\overline{\phi}$ induces a bounded Hankel operator on the Hardy space (or, equivalently, if $\overline{\phi} \in \mathrm{BMOA}$, in which case $||\overline{\phi}||_{BMO} \sim ||S_\phi||$) [**10**], then with respect to the basis $\{z^n\sqrt{n+1}, n \geq 0\}$, the matrix of $S_\phi \in B(\mathcal{L}_a^2)$ is $[a_{i+j}m_{ij}]$ where $[a_{i+j}]$ is the matrix of $S_\phi \in B(H^2)$with respect to the basis $\{e^{in\theta}, n \geq 0\}$ and $m_{ij} = \sqrt{(i+1)(j+1)}/(i+j+1)$.

This brings us to the map Φ which sends bounded Hankel operators on H^2 into bounded Bergman-Hankel operators on $\mathcal{L}_a^2$ via Schur multiplication defined by $\Phi(A) = [a_{ij}m_{ij}]$ where $A = [a_{ij}]$ with respect to the standard basis $\{e^{in\theta}, n \geq 0\}$ of H^2 and m_{ij} is the multiplier defined above.

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