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INTERPOLATION OF SPECTRUM OF BOUNDED OPERATORS ON LEBESGUE SPACES

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ABSTRACT. Let μ be a σ-finite positive measure. Assume $1 \leq p < s < \infty$. Let T be a linear operator on $L^p(\mu) \cap L^s(\mu)$ that has bounded extensions T_p and T_s on $L^p(\mu)$ and $L^s(\mu)$ respectively. Then T has a bounded extension T_r on $L^r(\mu), p \leq r \leq s$. The aim of this paper is to study the relationship between the spectral and Fredholm properties of the operator T_r and those of T_p and T_s.

1. Introduction. Let μ be a fixed positive σ-finite measure, and let $L^p = L^p(\mu)$ be the usual Lebesgue spaces relative to μ for $1 \leq p \leq \infty$. Assume $1 \leq p < s < \infty$. Suppose T is a linear operator mapping $L^p \cap L^s$ into itself such that T has bounded extensions T_p on L^p and T_s on L^s. Then the Riesz Convexity Theorem [**7**; Theorem 11, p. 525] implies that, for $p < r < s, T$ has a bounded extension T_r on L^r with

$$||T_r|| \leq \max\{||T_p||, ||T_s||\}.$$

Let $\sigma(T)$ denote the spectrum of an operator T. It is not difficult to find examples where $\sigma(T_r)$ is different for different $r \in [p, s]$; see for example [**6**] or [**10**, pp. 328–329].

One aim of this paper is to deal with the following questions in the situation described above:

(i) How does $\sigma(T_r)$ relate to $\sigma(T_p)$ and $\sigma(T_s)$?

(ii) If T_p and T_s are Fredholm operators, then under what conditions is T_r a Fredholm operator?

(iii) How does the Fredholm spectrum and the Weyl spectrum of T_r relate to the same spectra of T_p and T_s?

Some answers to these questions are given in §4 and §5. (The case where $s = \infty$ is also included.) Question (i) has been considered by a number of mathematicians; see [**2**], [**8**], [**9**], and [**15**].

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