ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 20, Number 2, Spring 1990

TENSOR PRODUCTS OF NON-SELF-ADJOINT OPERATOR ALGEBRAS

V.I. PAULSEN* AND S.C. POWER**

1. Introduction. In this paper we study several norms that can be introduced on the algebraic tensor product of two, not necessarily self-adjoint, algebras of operators on a Hilbert space.

Following the work of Arveson [**2**], we know that if $\mathcal{A}$ is an algebra of operators on a Hilbert space $\mathcal{H}$ or, more generally, a subalgebra of a C^*-algebra $\mathcal{B}$, then to fully understand $\mathcal{A}$ we must also consider the whole family of norms on the k by k matrix algebras over $\mathcal{A}, \mathcal{M}_k(\mathcal{A})$. That is, we must regard $\mathcal{A}$ as a *matrix normed* space in the sense of Effros [**4**]. When $\mathcal{A}$ is an algebra of operators on $\mathcal{H}$, then $\mathcal{M}_k(\mathcal{A})$ is just the algebra of $k \times k$ matrices with entries from $\mathcal{A}$. This can be regarded as an algebra of operators on $\mathcal{H} \oplus \cdots \oplus \mathcal{H}$ (k times), denoted $\mathcal{H}^{(k)}$, and is endowed with the norm that it inherits as operators on $\mathcal{H}^{(k)}$. When $\mathcal{A}$ is a subalgebra of a C^*-algebra $\mathcal{B}$, then it is well-known that there is a unique norm on $\mathcal{M}_k(\mathcal{B})$ which makes it into a C^*-algebra, and we endow $\mathcal{M}_k(\mathcal{A})$ with the norm that it inherits as a subspace.

For the above reasons, if we are given an arbitrary complex algebra $\mathcal{A}$, then we shall call $\mathcal{A}$ an *operator algebra*, if it is endowed with a family of norms on $\mathcal{M}_k(\mathcal{A})$ and a representation ρ of $\mathcal{A}$ on some Hilbert space such that the norms on $\mathcal{M}_k(\mathcal{A})$ are induced by the representation. Thus

$$||(a_{ij})|| = ||(\rho(a_{ij}))||$$

for all (a_{ij}) in $\mathcal{M}_k(\mathcal{A})$ and all k. We call such a family of norms an *operator norm*.

Given two unital operator algebras, $\mathcal{A}_1$ and $\mathcal{A}_2$, we define a *complete operator cross-norm* to be any operator norm on $\mathcal{A}_1 \otimes \mathcal{A}_2$ which is a cross-norm, that is, $||a_1 \otimes a_2|| = ||a_1|| \cdot ||a_2||$, and which has the

Supported in part by an NSF grant.
Supported by a D.G. Bourgin visiting Scholarship and the Science and Engineering Research Council.
Received by the editors on November 19, 1987 and, in revised form, on December 14, 1987.