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RATIONAL C*-ALGEBRAS AND NONSTABLE K-THEORY

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1. Introduction. In this article, we will be concerned with various aspects of comparison theory for idempotents (or finitely generated projective modules) over unital rings, particularly over unital $\mathbf{C}^*$-algebras.

The usual way equivalence and comparison is studied is via K-theory, which involves "stabilizing" the relations. There is a very extensive and powerful body of machinery allowing the computation of the K-groups of many rings; the problem is then to relate the K-theory data back to the actual structure of the ring and its projective modules. This process has become known as *nonstable K-theory*.

We will discuss several aspects of nonstable K-theory and develop some new relationships based on results in the theory of abelian semigroups. Then we will apply these results to solve nonstable K-theory problems for certain "rationalized" rings. Actually, a majority of the paper is devoted to developing the semigroup theory; in fact, a more appropriate title might be "applications of abelian semigroups in algebraic K-theory."

1.1 *Review of K_0-Theory.* Let us first give a very brief review of the construction of $K_0(A)$ for a unital ring A, in order to establish notation. A much more complete treatment of the subject can be found in [**3**].

Definition 1.1.1. Let p and q be idempotents in A. $p \sim q$ if there are $x, y \in A$ with $xy = p, yx = q$. $p \prec q$ if p is equivalent to an idempotent r with $qr = rq = q$ and $r \neq q$. $p \preceq q$ if $p \prec q$ or $p \sim q$. A is *finite* if $1_A \not\prec 1_A$; A is *stably finite* if the $n \times n$ matrix algebra $M_n(A)$ is finite for all n. $M_\infty(A)$ is the (nonunital) ring $\varinjlim M_n(A)$, where $M_n(A)$ is embedded in $M_{n+1}(A)$ in the upper left-hand corner, extended by zeros.

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