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INNER MULTIPLIERS OF THE BESOV SPACE, $0 < p \leq 1$

PATRICK AHERN AND MIROLJUB JEVTIĆ

0. For $\alpha > 0$ let k be the integer so that $k-1 \leq \alpha < k$. Then, for $p > 0$, the Besov space B^p_α is the set of functions f, holomorphic in the unit disc U such that

$$||f||^p_{p,\alpha} = \int |f^{(k)}(z)|^p (1-|z|)^{p(k-\alpha)-1} dm(z) < \infty.$$

Here dm denotes area measure in U. We will assume from now on that $1 - p\alpha > 0$. (When $1 - p\alpha < 0$ the functions in B^p_α are continuous out to the boundary of U.) In [**9**], I. Verbitsky characterized those inner functions $B \in MB^p_\alpha$, i.e., for which $Bf \in B^p_\alpha$ for all $f \in B^p_\alpha, p \geq 1$. See [**5**, Chapter 17], for a discussion of inner functions. In this paper we consider the case $0 < p \leq 1$.

The first step is to show that any such inner function is a Blaschke product whose zero set is a finite union of interpolating sequences. The proof of this for $p \leq 1$ is similar to Verbitsky's proof for $p \geq 1$. Indeed, after some preliminaries we appeal directly to his argument. So the question becomes: Which such Blaschke products are in MB^p_α?

For $p > 1$, the Carleson measures for B^p_α were determined by D. Stegenga [**6**]. Using this result one immediately gets a necessary and sufficient condition on B in order that $B \in MB^p_\alpha$. However, this condition does not involve the distribution of zeros of B in any direct way. The whole point of Verbitsky's paper is to find a necessary and sufficient condition on the zeros of B in order that $B \in MB^p_\alpha$. We take the same point of view.

In the first section we find the Carleson measures for $B^p_\alpha, 0 < p \leq 1$. For the case $p > 1$, Stegenga used the ideas involved in E. Stein's proof [**7**] of the original Carleson measure theorem together with the strong capacitary estimates of D. Adams [**1**]. Our proof is the same except we must use the recently proved "strong Hausdorff capacity" estimates

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