MICROBIAL GROWTH IN PERIODIC GRADOSTATS

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Dedicated to the memory of Geoffrey Butler.

Introduction. A gradostat is a laboratory device in which one can study the growth of microorganisms in a nutrient gradient. As constructed by Lovitt and Wimpenny [**7, 8, 16**], a gradostat is a concatenation of several chemostats in which adjacent vessels are connected by tubes allowing pumps to exchange the material contents of each vessel. A mathematical model of the growth of a single species of microorganism in an n-vessel gradostat based on Michaelis-Menten growth response was studied by Tang in [**15**]. A model of competition between two different species in a two-vessel gradostat was studied by Jäger, So, Tang and Waltman in [**5**]. A similar model was studied by the author together with Tang in [**12**]. Models of competition between two species of microorganisms in a gradostat-like device have been studied by Stephanopoulos and Fredrickson [**13**] and Kung and Baltzis [**6**] in the bioengineering literature.

Actually, the term gradostat does not refer to a single well-defined apparatus, but to a whole family of configurations of interconnected, well-stirred vessels in which one can study the growth of microorganisms. We refer the reader to [**16**] for a general discussion of gradostat devices. See, in particular, Figures 1 and 10 describing different possible configurations.

The aim of the present paper is to study the growth of a single species of microorganism in the presence of one limiting substrate or two limiting complementary substrates in a very general gradostat where we allow essentially arbitrary connections between vessels, outside feed reservoirs of a limiting nutrient, and receiving vessels. In addition, we allow operating parameters, e.g., flow rates, input limiting nutrient concentration, to be time varying in a periodic way. Allowing periodic operating parameters is not merely a mathematical exercise. Long period variations of operating parameters can simulate noise, inevitably

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