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MONOTONICITY PROPERTIES OF THE MICHAELIS-MENTEN REACTIONS OF ENZYME KINETICS

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ABSTRACT. The Michaelis-Menten reactions of enzyme kinetics can be written $E + S \overset{\rightarrow}{\leftarrow} ES \rightarrow P + E$ (E is the enzyme). Assuming mass-action kinetics, the concentrations are governed by a system of ordinary differential equations. An investigation is made of the signs of the derivatives of the concentrations with respect to each initial concentration. Although the system does not give rise to an order preserving flow with respect to an orthant, many of the derivatives with respect to an initial concentration are of one sign.

1. Introduction. The Michaelis-Menten reactions of enzyme kinetics can be written

$$E + S \underset{k_{-1}}{\overset{k_1}{\rightleftarrows}} ES \overset{k_2}{\rightarrow} P + E,$$

where E, S, ES, and P are the enzyme, substrate, complex, and product, respectively. Denoting the concentrations of E, S, ES, and P by x, y, z, w, respectively, the law of mass-action gives the system of differential equations

$$\begin{aligned} \dot{x} &= -k_1 xy + (k_{-1} + k_2)z \\ \dot{y} &= -k_1 xy + k_{-1} z \\ \dot{z} &= k_1 xy - (k_{-1} + k_2)z \\ \dot{w} &= k_2 z \end{aligned} \quad , \tag{1.1}$$

where a dot indicates the time derivative and k_1, k_{-1}, and k_2 are positive constants (the rate constants). For background, see [**5**] and [**3**].

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