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# LIMITS OF HETEROCLINC ORBITS IN A COMPETITIVE MODEL WITH GENETIC VARIATION

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ABSTRACT. A three-dimensional system of autonomous ordinary differential equations which models the competition between two populations with genetic variation in one population is studied. The competitive interaction is of Lotka/Volterra type. On one allele frequency fixation plane the dynamical behavior is that of stable coexistence and, on the other, mutual exclusion. There are heteroclinic orbits connecting the two fixation planes, and the equilibria which these orbits approach vary depending on the crowding parameter of the genetically invariant population. For a critical value of this parameter, there is a line of polymorphic equilibria. It is shown that portions of this line along with another heteroclinic orbit form the topological limit of the orbits connecting the fixation planes as the parameter approaches its critical value. Hence, this provides a better understanding of the heteroclinic bifurcation occurring at the critical value of the parameter.

**1. Introduction.** Competition between two populations may be modeled by the two-dimensional system of ordinary differential equations,

$$\begin{aligned} \dot{M} &= \mu(M,N)M \\ \dot{N} &= \eta(M,N)N, \end{aligned} \tag{1}$$

where $M, N \geq 0$ are the population sizes (or densities) and $\mu, \eta$ are $C^1$ functions with $\partial\mu/\partial N < 0$ and $\partial\eta/\partial M < 0$. The functions $\mu$ and $\eta$ are per capita growth rates for the $M$ and $N$ populations, respectively. We refer to $\mu$ and $\eta$ as *fitness* functions. The competition is said to be Lotka/Volterra if the fitnesses are linear functions of $M$ and $N$, i.e.,

$$\begin{aligned} \mu(M,N) &= r_M - \alpha M - \beta N \\ \eta(M,N) &= r_N - \delta M - \gamma N, \end{aligned} \tag{2}$$

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