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PERIODIC PERTURBATIONS OF LINEAR PROBLEMS AT RESONANCE ON CONVEX DOMAINS

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ABSTRACT. We consider Dirichlet problems for semilinear elliptic equations whose nonlinear term is periodic and whose linear part is resonance. We show that such problems have infinitely many positive and infinitely many negative solutions on domains in the plane which are convex. The arguments used do not carry over to dimension greater than three. This work complements some earlier work of ours.

1. Introduction. Let Ω be a bounded domain in $\mathbf{R}^n$ with smooth boundary. As is well known, the principle eigenvalue λ_1 of the Dirichlet problem

$$\begin{gathered}\Delta u+\lambda u=0, \quad x\in\Omega,\\ u=0, \quad x\in\partial\Omega,\end{gathered} \tag{1}$$

is simple and has an associated eigenfunction ϕ with the properties

$$\phi(x)>0, \quad x\in\Omega, \quad \frac{\partial\phi(x)}{\partial\nu}<0, \quad x\in\partial\Omega,$$

where $\partial/\partial\nu$ is the exterior normal derivative to $\partial\Omega$. (We normalize ϕ so that $\phi_{\max}=1$.)

In this paper we consider the *resonant* nonlinear problem

$$\begin{gathered}\Delta u+\lambda_1 u+g(u)=h(x), \quad x\in\Omega,\\ u=0, \quad x\in\partial\Omega,\end{gathered} \tag{2}$$

where $h:\overline{\Omega}\to\mathbf{R}$ and $g:\mathbf{R}\to\mathbf{R}$ are Hölder continuous functions and satisfy

$$\int_\Omega h\phi\,dx=0, \tag{3}$$

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