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EXISTENCE OF TRANSVERSAL HOMOCLINIC POINTS IN A DEGENERATE CASE

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Introduction. Let F be a diffeomorphism on a compact manifold. Smale [**8, 9**] shows that if F has a transversal homoclinic point there is a Cantor-like set on which some iterate of F is invariant and isomorphic to the Bernoulli shift on a finite number of symbols. In Palmer [**5, 7**] it was shown how this result could be simply deduced from the shadowing lemma for hyperbolic sets.

Also, in [**5**] (see also Gruendler [**2**]), a periodic differential equation

$$\dot{x} = g(x) + \mu h(t, x, \mu), \quad x \in \mathbf{R}^k, \tag{1}$$

was considered, where the unperturbed system

$$\dot{x} = g(x) \tag{2}$$

has a saddle point and an associated homoclinic connection $\phi(t)$ such that, up to a scalar multiple, $\phi'(t)$ is the unique bounded solution of the variational equation

$$\dot{x} = g'(\phi(t))x. \tag{3}$$

Under this condition the equation adjoint to (3) also has, up to a scalar multiple, a unique bounded solution $\psi(t)$, and, if the Melnikov function

$$\Delta(\alpha) = -\int_{-\infty}^{\infty} \psi^*(t+\alpha)h(t, \phi(t+\alpha), 0)\, dt$$

has a simple zero, it turns out for $\mu \neq 0$ sufficiently small that the period map for equation (1) has a transversal homoclinic point.

Note that the condition on the variational equation (3) is equivalent to the requirement that the tangent spaces to the stable and unstable manifolds of the saddle point have a one-dimensional intersection along the homoclinic orbit $\phi(t)$. In this paper we want to relax this condition,