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ASYMPTOTIC AND OSCILLATORY BEHAVIOR OF A CLASS OF SECOND ORDER FUNCTIONAL DIFFERENTIAL EQUATIONS

B. S. LALLI AND S. R. GRACE

Dedicated to the memory of Professor G. J. Butler

ABSTRACT. The asymptotic and oscillatory behavior of solutions of functional differential equations of the form

$$(a(t)\psi(x(t))\dot{x}(t))^{\cdot} + p(t)\dot{x}(t) + q(t)f(x[g(t)]) = 0, \quad \left(\cdot = \frac{d}{dt}\right),$$

is discussed. Here q is allowed to change sign on $[t_0, \infty)$.

1. Introduction. Consider the functional differential equations of the type

$$(1.1) \quad (a(t)\psi(x(t))\dot{x}(t))^{\cdot} + p(t)\dot{x}(t) + q(t)f(x[g(t)]) = 0, \quad \left(\cdot = \frac{d}{dt}\right),$$

where $q, g, p, q : [t_0, \infty) \to \mathbf{R}$, $\psi, f : \mathbf{R} \to \mathbf{R}$ are continuous, $a(t) > 0$, $q(t) \geq 0$, and q is not identically zero on any subinterval of $[t_0, \infty)$. Moreover, $g(t) \to \infty$ as $t \to \infty$, $\psi(x) > 0$ for all x and $xf(x) > 0$ for $x \neq 0$, and functions q, g, a and ψ are continuously differentiable.

In what follows, we consider only such solutions which are defined for all $t \geq t_0 \geq 0$. The oscillatory character is considered in the usual sense; i.e., a continuous real-valued function x defined on $[t_x, \infty)$, for some $t_x \geq 0$, is called oscillatory if its set of zeros is unbounded above, otherwise it is called nonoscillatory.

In recent years there has been an increasing interest in the study of the qualitative behavior of solutions of equations of type (1.1). Kulenovic and Grammatikopoulos [**13**] obtained some results on the behavior of the retarded strongly superlinear equation

$$(*) \qquad (a(t)\dot{x}(t)) + q(t)f(x[q(t)]) = 0.$$