

OSCILLATIONS OF DIFFERENCE EQUATIONS WITH POSITIVE AND NEGATIVE COEFFICIENTS

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ABSTRACT. We obtain sufficient conditions for the oscillation of all solutions of some difference equations with positive and negative coefficients. Our results include the following: Consider the difference equation

$$(1) \quad A_{n+1} - A_n + pA_{n-k} - qA_{n-l} = 0, \quad n = 0, 1, 2, \dots,$$

where p and q are nonnegative real numbers and k and l are nonnegative integers such that

$$p > q \geq 0, \quad k \geq l \geq 0, \quad q(k-l) \leq 1$$

and

$$p - q > \frac{k^k}{(k+1)^{k+1}} \quad \text{if } k \geq 1$$
$$p - q \geq 1 \quad \text{if } k = 0.$$

Then every solution of Equation (1) oscillates. Extensions to equations with variable coefficients were also obtained.

1. Introduction and preliminaries. Recently, Györi and Ladas [5], Ladas [7] and Erbe and Zhang [3] investigated the oscillatory behavior of solutions of difference equations of the form

$$(1) \quad A_{n+1} - A_n + \sum_{j=0}^m P_j(n)A_{n-j} = 0, \quad n = 0, 1, 2, \dots,$$

with positive coefficients $P_j(n)$. Our aim in this paper is to obtain oscillation results for some difference equations with positive and negative coefficients.

Let $\mathbf{N} = \{0, 1, 2, \dots\}$ be the set of natural numbers and Δ denote the forward difference operator defined by $\Delta A_n = A_{n+1} - A_n$. Consider the linear difference equation with positive and negative coefficients

$$(2) \quad \Delta A_n + P(n)A_{n-k} - Q(n)A_{n-l} = 0, \quad n = 0, 1, 2, \dots,$$

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