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THE EXISTENCE OF AN EQUILIBRIUM FOR PERMANENT SYSTEMS

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ABSTRACT. The criterion of permanence for biological systems requires that there exist a compact attractor for the interior of the positive cone X lying in $\operatorname{int} X$. It is shown here that for several models permanence implies the existence of an equilibrium point in $\operatorname{int} X$ corresponding to a stationary coexistence state.

1. Introduction. The criterion of permanence for biological systems is a condition ensuring the long-term survival of all species. Sufficient conditions for permanence have been given for a wide variety of models, see, for example, [**3, 4, 5, 7, 8, 10, 11, 12, 13**]. To illustrate the question to be tackled here, consider a model based on a system of autonomous ordinary differential equations

$$\dot{x}_i = x_i f_i(x), \qquad i = 1, \dots, n, \tag{1}$$

on the positive cone $\mathbf{R}^n_+$, where $x = (x_1, \dots, x_n)$ and conditions ensuring the global existence and uniqueness of solutions in forward time are imposed. The system (1) is said to be *permanent* if there exist $m, M \in (0, \infty)$ such that, given any $x \in \operatorname{int} \mathbf{R}^n_+$, there is a t_x such that

$$m \leq x_i(t) \leq M, \qquad i = 1, \dots, n, t \geq t_x.$$

From a biological point of view, it is reasonable to expect that if permanence holds, there will be a stationary coexistence state in $\operatorname{int} \mathbf{R}^n_+$. If such a state does exist, a natural necessary condition for permanence follows. An analogous question may be asked for the system of difference equations

$$x_i' = x_i f_i(x), \qquad i = 1, \dots, n, \tag{2}$$

where x_i' denotes the value of x_i at the next generation. As has been noted, for example, in [**8**] and [**10**], the question for both these systems has an affirmative answer. The methods of proof have often been