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POSITIVE SOLUTIONS OF A BOUNDARY VALUE PROBLEM

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For the moment, let $\mathcal{K}$ be a cone in $\mathbf{R}^n$. Then it is easy to prove that if

$$-u''(t) \in \mathcal{K}, \qquad t \in [a,b],$$
$$u(a) \in \mathcal{K}, \qquad u(b) \in \mathcal{K},$$

then $u(t) \in \mathcal{K}$ for $t \in [a,b]$. This result was used in the work of Schmitt and Smith [**3**] on extremal solutions. Our main goal is to prove a generalization of this result.

First, we give some preliminary definitions and results. Let $\mathcal{X}$ be a Banach space. A closed subset $\mathcal{K} \subseteq \mathcal{X}$ is said to be a *cone* provided

(i) if $u, v \in \mathcal{K}$, then $\alpha u + \beta v \in \mathcal{K}$ for all $\alpha, \beta \geq 0$,

(ii) if $u, -u \in \mathcal{K}$, then $u = \theta$ (the zero element of $\mathcal{X}$).

A cone $\mathcal{K}$ is *solid* if its interior $\mathcal{K}^\circ \neq \varnothing$. As in [**2**], if $u, v \in \mathcal{X}$, we write $u \leq v$ in case $v - u \in \mathcal{K}$, and we write $u \ll v$ in case $v - u \in \mathcal{K}^\circ$.

LEMMA 1. *Let $\mathcal{K}$ be a cone in a Banach space $\mathcal{X}$. If $y(t)$ is the solution of the boundary value problem*

$$\begin{aligned} y^{(n)}(t) &= \theta, \qquad t \in [a,b],\\ y^{(i)}(a) &= \theta, \qquad 0 \leq i \leq k-1,\\ y^{(i)}(b) &= \theta, \qquad 0 \leq i \leq n-k-1,\ i \neq j,\\ (-1)^j y^{(j)}(b) &= \beta_j \in \mathcal{K}, \end{aligned}$$

where j is a fixed integer with $0 \leq j \leq n-k-1$, then

$$y(t) \in \mathcal{K}, \qquad t \in [a,b].$$