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RIGHT AND LEFT DISCONJUGACY IN DIFFERENCE EQUATIONS

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1. Introduction. We are concerned with the n-th order difference equation

$$(1) \qquad Py(t) \equiv \sum_{i=0}^{n} \alpha_i(t) y(t+i) = 0, \quad t \in [a,b],$$

where $a < b$ are integers and $[a,b] \equiv \{a, a+1, \dots, b\}$, $\alpha_n = 1$, and α_0 satisfies

$$(2) \qquad (-1)^n \alpha_0(t) > 0, \quad t \in [a,b].$$

Solutions of the difference equation (1) are defined on $[a, b+n]$.

In part, we will be concerned with a partial factorization of P if (1) is right $(j, n-j)$-disconjugate. In addition, we give several results relating right and left disconjugacy and disconjugacy.

As defined by Hartman, (1) is said to be disconjugate on an interval J if no nontrivial solution has n generalized zeros on J. In the classic paper [**2**], Hartman has shown that (1) is disconjugate on J if and only if P has a certain factorization. Further, necessary and sufficient conditions for disconjugacy in terms of the coefficients $\alpha_i(t)$ are given, and sign conditions on the Green's functions for certain boundary value problems for a disconjugate difference equation are given.

More recently, Peterson [**5**] defined the more general notions of right and left disconjugacy. Necessary conditions for right $(j, n-j)$-disconjugacy in terms of the coefficients $\alpha_i(t)$ are given in [**4**]. Peterson [**7**] also gave necessary and sufficient conditions for $(j, n-j)$-disconjugacy in terms of certain Wronskians. Finally, Peterson [**6**] gave sign conditions on the Green's functions for boundary value problems where (1) satisfies certain $(j, n-j)$-disconjugacy conditions.

2. Preliminaries. Define the difference operator Δ by $\Delta y(t) = y(t+1) - y(t)$, and define the operators Δ^i by $\Delta^i y(t) = \Delta(\Delta^{i-1} y(t))$