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HOMOGENEOUS MODELS FOR SEXUALLY TRANSMITTED DISEASES

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ABSTRACT. A system of eight ordinary differential equations describes birth, death, formation of pairs, separation, and transmission of a sexually transmitted disease. Here, in contrast to an earlier version of the model by Dietz and Hadeler, the recruitment process is coupled to the actual population size. Nevertheless, as in most demographic models, the equations are assumed homogeneous. There is a noninfected exponentially growing persistent solution which is stable (in the sense of the stability theory for homogeneous equations) for low rates of pair formation and low infectivity. If these parameters are increased, this state may lose stability, a stable persistent solution describing an infected population bifurcates. The exact bifurcation thresholds are derived in terms of the epidemiologically relevant parameters.

1. Introduction. In several recent publications Dietz and Hadeler [5, 6] have discussed a model for sexually transmitted diseases. (For other recent work in this direction see [1, 4, 7, 14, 15]. In this model the social structure is taken into account. Since a strict pair is practically temporarily immune against infection, in a population with a large number of pairs, the spread of the disease is much slower than in a population with random sexual contacts. In these papers it has been assumed that the process of recruitment of young individuals acts on a much slower time scale than the infection process. Thus it has been assumed that the population is renewed with a constant rate, independent of the actual population size. But in some diseases the time scale of the epidemic process has the same order of magnitude as the demographic processes (see also Anderson et al. [1]), and the spread of the disease has a marked impact on the demographic evolution. In the present work we consider essentially the same model, but we assume that the number of newly recruited individuals is proportional to the actual population size. As in Dietz and Hadeler [6], we obtain a system of eight ordinary differential equations for the different classes of individuals. In this case the right-hand side of the differential system

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