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ASYMPTOTIC CONDITIONS FOR THE SOLVABILITY OF A FOURTH ORDER BOUNDARY VALUE PROBLEM WITH PERIODIC BOUNDARY CONDITIONS

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ABSTRACT. This paper concerns the existence of solutions of the fourth order periodic boundary value problem

$$-\frac{d^4u}{dx^4} + f(u(x))u'(x) + g(x, u(x)) = e(x), \quad x \in [0, 2\pi],$$
$$u(0) - u(2\pi) = u'(0) - u'(2\pi)$$
$$= u''(0) - u''(2\pi) = u'''(0) - u'''(2\pi) = 0,$$

under some nonuniform resonance and nonresonance conditions on the asymptotic behavior of $u^{-1}g(x, u)$ for $|u| \to \infty$.

1. Introduction. Fourth order boundary value problems arise in the study of the equilibrium of an elastic beam under an external load (e.g., see [**1, 2, 5, 6, 16**]), where the existence, uniqueness and iterative methods to construct the solutions have been studied extensively. The author studied in [**7**] the following fourth order boundary value problems with periodic boundary conditions:

$$\frac{d^4u}{dx^4} + f(u)u' + g(x, u) = e(x), \quad x \in [0, 2\pi],$$
$$u(0) - u(2\pi) = u'(0) - u'(2\pi) = u''(0) - u''(2\pi)$$
$$= u'''(0) - u'''(2\pi) = 0$$

(1.1)

and

$$-\frac{d^4u}{dx^4} + \alpha u' + g(x, u) = e(x), \quad x \in [0, 2\pi],$$
$$u(0) - u(2\pi) = u'(0) - u'(2\pi) = u''(0) - u''(2\pi)$$
$$= u'''(0) - u'''(2\pi) = 0,$$

(1.2)

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