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NUMERICAL EXTREMAL METHODS AND BIOLOGICAL MODELS

JOHN GREGORY[1], CANTIAN LIN[1], and RONG SHENG WANG

ABSTRACT. Discrete variable methods are given to find extremaloid solutions (extremals with corners) in the calculus of variations and optimal control theory for well defined mixtures of initial value problems and boundary value problems. Our methods are general, efficient, and accurate with a global a priori, pointwise error of $O(h^2)$ and a Richardson error of $O(h^4)$. Our methods are motivated by a generalization of Henrici's methods for ordinary differential equations and our discretized equations are tridiagonal, which is very important in practical applications.

1. Introduction. A major chapter in classical applied mathematics is to find the minimum of an integral of the form

$$I(x) = \int_a^b f(t, x, x')\, dt \tag{1}$$

subject to conditions which yield a unique solution. While there are many necessary and sufficient conditions, the first and major condition is that the minimizing solution $x(t)$ satisfies the Euler-Lagrange equation

$$\frac{d}{dt} f_{x'} = f_x \tag{2}$$

between corners on $[a, b]$. In this case, $x(t)$ is called an extremaloid solution (see [**4**, p. 60]). This equation follows from the first variational equation which requires that $x(t)$ satisfy

$$I'(x, y) = \int_a^b [f_x y + f_{x'} y']dt = 0 \tag{3}$$

for all admissible variations $y(t)$. Similarly, closely related problems (defined below) occur in the field of optimal control theory.

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