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EXIT PROBABILITIES FOR STOCHASTIC POPULATION MODELS: INITIAL TENDENCIES FOR EXTINCTION, EXPLOSION, OR PERMANENCE

THOMAS C. GARD

ABSTRACT. Let $X(t) = X(t,\omega)$ be a stochastic process which represents the (ω-th sample) population density at time t, $t \geq 0$ ($\omega \in \Omega$, a sample space), and let P be the underlying probability measure (defined on Ω). Let $L < U$ be positive constants, and fix an initial population distribution $X(0)$ satisfying $P(L < X(0) < U) = 1$; the population density levels L and U may correspond to effective extinction and explosion respectively, for the population, for example. Denote by $\tau = \tau(\omega)$ the first exit time of X from the interval $(L,U) : \tau = \inf\{t : X(t) \notin (L,U)\}$. The probabilities $P(\tau = +\infty), P(\tau < +\infty, X(\tau) \geq U)$, and $P(\tau < +\infty, X(\tau) \leq L)$ represent the permanence probability, and the initial tendencies of the population toward explosion and extinction respectively relative to the interval $[L,U]$. These probabilities are calculated for some diffusion process models. A result is given which shows that initial tendencies not to explode or go extinct for diffusion process models follow from dissipativeness or persistence for associated deterministic models respectively.

1. Introduction. The exponential population growth model corresponding to the assumption of constant per capita net growth rate is generally rejected since it predicts that population densities become unbounded. Indeed, boundedness of solutions, as a crudest form of stability, is usually expected of dynamical system models of population evolution, since this type of behavior is observed in real populations. Thus, checking that solutions are bounded constitutes an important step in validating a specific model. Once the boundedness question is answered, qualitative considerations such as persistence can be addressed: Does the model predict that the population(s) will survive indefinitely? When environmental or demographic variability are accounted for via stochastic models, a number of interpretations of boundedness and persistence are possible. Exactly how such variability as expressed by stochastic models effects qualitative behavior is as yet unsettled (Chesson [1, 2, and 3], Murdoch [14], for example). Tractable

Key words. Stochastic population models, transient behavior.