A SURVEY OF G.J. BUTLER'S RESEARCH
IN THE QUALITATIVE THEORY OF
ORDINARY DIFFERENTIAL EQUATIONS

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1. Introduction. In this paper we shall survey some of the main results obtained by our late colleague and friend, Professor Geoffrey James Butler. The outline is as follows. We shall review his work on periodic solutions for nonlinear differential equations (§2), oscillation and nonoscillation for second order linear and nonlinear equations (§3), comparison theorems and oscillation for higher order linear equations and systems (§4). In §5, we discuss some miscellaneous contributions in chaotic behavior of mappings and fixed point theorems and packing and covering problems.

2. Periodic solutions of second order nonlinear equations. Consider the second order nonlinear differential equations

$$(2.1) \qquad x'' + f(x)h(x'^2) + g(x) = \mu p(t)$$

and

$$(2.2) \qquad x'' + f(x)h(x'^2) + g(x) = 0,$$

where f, g, h and p are continuous with p periodic of period ω and μ is a parameter. Extending earlier results of Heidel [**A10**] and Utz [**A20**], Butler in [**9**] gave necessary and sufficient conditions for the existence of infinitely many periodic solutions. Furthermore, he also obtained sufficient conditions for the existence of periodic solutions of (2.2) with arbitrarily large periods. These are summarized in

THEOREM 2.1. [**9**]. *Let h be locally Lipschitz in a neighborhood of zero. Then (2.2) has infinitely many periodic solutions if and only if property* (P) *below holds*: