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MEROMORPHIC EXTENSION OF ANALYTIC CONTINUED FRACTIONS ACROSS THE LINE OF NONCONVERGENCE

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Dedicated to Professor Wolfgang J. Thron
on the occasion of his 70-th birthday

ABSTRACT. Continued fractions $K(-a_n(z)/\lambda(z))$ are considered, where $a_n(z) \not\equiv 0$, $n \in \mathbf{N}$, and $\lambda(z)$ are holomorphic functions on a region $M \subset \mathbf{C}$ such that $\lim_{n\to\infty} a_n(z) = 1/4$ holds uniformly in M. They converge on $M \backslash S$, $S := \{z \in M : \lambda(z) \in [-1,1]\}$, to a meromorphic function $F(z)$. Conditions on the speed of convergence of the sequence $a_n(z)$, $n \in \mathbf{N}$, are given which ensure that $F(z)$ can be extended meromorphically across S into a part of the Riemann-surface of $\lambda(z) - (\lambda^2(z) - 1)^{1/2}$. For special classes of continued fractions, explicit analytic extension results are given.

1. Introduction and main results. We first consider limit-periodic analytic continued fractions of the type

$$(1) \qquad f(\lambda) = \frac{1}{\lambda} \,\underset{-}{}\, \frac{a_1}{\lambda} \,\underset{-}{}\, \frac{a_2}{\lambda} \,\underset{-}{}\, \frac{a_3}{\lambda} \,\underset{-}{}\, \cdots,$$

where $a_n \in \mathbf{C}$, $a_n \neq 0$ for all $n \in \mathbf{N}$ and $\lim_{n\to\infty} a_n = 1/4$ holds. It is well known (see [**3, 4, 8**]), that the right side in (1) converges and represents a meromorphic function $f(\lambda)$ in $D^* := \mathbf{C}\backslash[-1,1]$, the complex plane with a cut along $[-1,1] \subset \mathbf{R}$. Let D^{**} be a second copy of D^* and assume that D^* and D^{**} are connected along the cut $[-1,1]$ by crosswise joining opposite boundaries of the cut. This generates the Riemann surface of $(\lambda^2-1)^{1/2}$ where $(\lambda^2-1)^{1/2} > 0$ for $\lambda > 1$, $\lambda \in D^*$.

In [**5**, Theorem 1] the author proved

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